Life Test Skip Lot Sampling Plans Based On Generalized Log-Logistic Model

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Abstract
Skip Lot acceptance sampling plan is proposed for the truncated life test based on product quality following generalized log-logistic distribution. For the proposed plan the minimum sample size necessary to ensure the specified median life are obtained at the given consumer’s confidence level. The operating characteristic values are analyzed, and the minimum ratios of the true population median life to the specified median life are also obtained at the specified producer’s risk. Selection and application of sampling plan is illustrated with a numerical example.

Keywords: Skip-Lot acceptance sampling plan, consumer’s confidence level, producer’s risk, operating characteristic function, binomial model.

Introduction
Acceptance sampling plans in statistical quality control are concerned with accepting or rejecting a submitted lot on the basis of the quality of the products inspected in a sample taken from that lot. If the quality of the product is the life time of the product then acceptance sampling plan becomes a life test plan. Products or items have variations in their lifetimes even though they are produced by the same producer, same machine and under the same manufacturing conditions. Due to this variation the producer and consumer are subject to risks. Increasing the sample size may minimize both risks to certain level but this will obviously increase the cost. To reduce these risks and cost an efficient acceptance sampling scheme with truncated life test is proposed.


With the introduction of modern quality management system such as ISO 9000, advocates lots from a statistically controlled process in which proportion of producing defective articles is constant, independent and very small. Under this situation it is feasible and desirable to use a skip-lot procedure, where inspection of certain lots may be skipped. Under this scenario Dodge and Perry(1973) [1] introduced Skip-lot sampling plan to achieve sampling economy. This motivated to design a Skip-lot sampling plan for life test in this paper.

This paper proposes the designing of a general Skip-Lot sampling plan for time-truncated life test based on generalized log-logistic distribution. The minimum sample sizes necessary to ensure the specified median life at the specified consumer’s confidence level are presented in section 2 using binomial model. Operating characteristic values are analyzed in section 3. Minimum median ratios are calculated for the specified producer’s risk in section 4. In section 5 a numerical example is provided to illustrate the selection of life test plan.

Generalized Log-Logistic Distributions
The lifetime of a product is assumed to have the generalized log-logistic distribution, whose probability density function and cumulative distribution function are given respectively as

\[ f(t;\sigma, \beta) = \frac{\beta (t/\sigma)^{\beta-1}}{1+(t/\sigma)^\beta}, \quad t \geq 0, \ \sigma > 0, \ \beta > 0, \ \theta > 0 \]  

(1)

and

\[ F(t;\sigma, \beta, \theta) = \left[ \frac{(t/\sigma)^\beta}{1+(t/\sigma)^\beta} \right]^\theta, \quad t \geq 0, \ \sigma > 0, \ \beta > 0, \ \theta > 0 \]  

(2)

where \( \sigma \) is the scale parameter \( \beta \) and \( \theta \) are shape parameters. The failure probability of a parallel system with \( \theta \) items having a log-logistic distributed lifetime is represented through cumulative distribution function. So, the generalized log-logistic distribution is considered to analyze the system reliability.
In a generalized log-logistic distribution the failure rate function is decreasing when $\beta \leq 1$ but increasing to certain level and then decreasing over time when $\beta > 1$. This distribution is adopted as a lifetime model in this study because its failure rate pattern is quite versatile. Therefore a skip-lot acceptance sampling plan for truncated life test under the generalized log-logistic distribution using median is proposed.

The median of the generalized log-logistic distribution is derived by

$$m = \sigma \left( \frac{0.5^{\theta/\sigma}}{1 - 0.5^{\theta/\sigma}} \right)^{1/\beta}$$  \hspace{1cm} (3)

Expression (3) shows that the median is proportional to the scale parameter, $\sigma$ when the shape parameters $\theta$ and $\beta$ are fixed. It is also seen that the median reduces to $m = \sigma$ regardless of $\beta$ when $\theta = 1$.

**Design Of The Proposed Skip Lot Sampling Plan**

Assume that the quality of a product can be represented by its median lifetime, $m$. Design of the Skip-Lot sampling plan for the truncated life test consists of determination of (i) sample size (ii) acceptance number (iii) the ratio of true median life to the specified median life $m/m_0$. The lot will be accepted if the submitted lot has a good quality when the experimental data supports the null hypothesis $H_0$: $m \geq m_0$ against the alternative hypothesis $H_1$: $m < m_0$ where $m_0$ is a specified lifetime.

The consumer risk, the probability of accepting a bad lot which has the true median life below the specified life $m_0$, is fixed as not to exceed $1 - P^*$.

Skip-lot sampling plan is followed under the assumption that there is a continuous flow of lots from the production process and lots are offered for inspection one by one in the order of production and the production process is capable of producing units whose process quality level is stable.

The operating procedure of Skip Lot sampling plan for the truncated life test has the following steps:

a) Start with normal inspection, using reference plan (single sampling plan is used as reference plan).

b) When $i$ consecutive lots are accepted on normal inspection, switch to skipping inspection and inspect only a fraction, $f$ of the lots.

c) When a lot is rejected on skipping inspection, switch to normal inspection.

d) Screen each rejected lot and correct or replace all defective units found.

Often it is convenient to set the termination time as a multiple of the specified lifetime $m_0$, in which case $t_0 = a m_0$ for a specified multiplier $a$. Then the proposed sampling plan is characterized by parameters $(n, c, f, i, a)$ where $i$ is an integer and $f$ is a fraction.

It is assumed that the lot size is large enough to use the binomial distribution to calculate the probability of acceptance of the lot.

The probability of acceptance of the lot for Skip lot sampling plan are

$$P_a = \frac{f P_i + (1-f) P_0}{f + (1-f)}$$  \hspace{1cm} (4)

Where

$$P = \sum_{x=0}^{c} n C_x p^x q^{n-x}$$  \hspace{1cm} (5)

In (5), $p$ is the probability that an item fails before $t_0$, which is given by

$$p = \left( \frac{0.5^{\gamma}}{1 - 0.5^{\gamma}} \right)^{1/\beta}$$  \hspace{1cm} (6)

with

$$\gamma = \left( \frac{0.5^{\theta/\sigma}}{1 - 0.5^{\theta/\sigma}} \right)^{1/\beta}$$  \hspace{1cm} (7)

The minimum sample size $n$ ensuring $m \geq m_0$ at the consumer’s confidence level $P^*$ can be found as the solution to the following inequality

$$P_a \leq 1 - P^*$$  \hspace{1cm} (8)

Multiple solutions exit for the sample size satisfying (8). To get unique values of the sample size $n$ the minimization of ASN function with the constraint is incorporated along with the specified consumer’s confidence level. Generally, the ASN for our skip-lot sampling plan is calculated by

$$\text{ASN} = F \cdot \text{ASN}(R)$$  \hspace{1cm} (9)
where \( F \) is average fraction inspected and \( \text{ASN}(R) \) is the average sample number of the reference sampling plan (single sampling plan).

Determination of the minimum sample size reduces to the following optimization problem

\[
\text{Minimize } \text{ASN} = \frac{nf}{f + (1-f)\beta} \quad (10)
\]

subject to \( P_s \leq 1-P \), \( n \geq 1 \) where \( n \) is an integer.

Minimum sample size may be obtained for various values of consumer confidence level (\( P^* \)) and the parameters \((i, f, a, c, \beta, \theta)\). Table (1) is constructed to give the minimum samples size for sample when \( P^* = 0.75, 0.90, 0.95, 0.99\), \( f=(0.25,0.333,0.50) \), \( i=3.0,1.2,3 \), \( a=(0.3, 0.4, 0.5, 0.6, 0.7, 1.1, 1.3) \), \( \beta=(1.2,3) \), \( \theta=(2.3) \) for the considered life test skip-lot sampling plan under generalized log-logistic distribution.

**TABLE (1). Minimum Sample Size And Asn Of Life Test Sksp-2 WITH i=3.**

<table>
<thead>
<tr>
<th>( (\beta, \theta) = (3, 2) ), ( f=0.50 )</th>
<th>( P^* )</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>0</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>27.1</td>
<td>17.1</td>
</tr>
<tr>
<td>0.9</td>
<td>43</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>42.8</td>
<td>27.9</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>36</td>
</tr>
<tr>
<td>0.9</td>
<td>54.9</td>
<td>35.2</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>98</td>
</tr>
<tr>
<td>0.9</td>
<td>85.5</td>
<td>54.3</td>
</tr>
<tr>
<td>9</td>
<td>84.9</td>
<td>54</td>
</tr>
<tr>
<td>0.7</td>
<td>9</td>
<td>99</td>
</tr>
<tr>
<td>0</td>
<td>53</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>51.3</td>
<td>32.2</td>
</tr>
<tr>
<td>0.9</td>
<td>8</td>
<td>91</td>
</tr>
<tr>
<td>0</td>
<td>73</td>
<td>47</td>
</tr>
<tr>
<td>5</td>
<td>72.8</td>
<td>46.3</td>
</tr>
<tr>
<td>0.9</td>
<td>88</td>
<td>57</td>
</tr>
<tr>
<td>0</td>
<td>87.9</td>
<td>56.4</td>
</tr>
<tr>
<td>5</td>
<td>78.9</td>
<td>56</td>
</tr>
<tr>
<td>0.9</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>0</td>
<td>122</td>
<td>79</td>
</tr>
<tr>
<td>5</td>
<td>121</td>
<td>78</td>
</tr>
<tr>
<td>0.9</td>
<td>99</td>
<td>99</td>
</tr>
</tbody>
</table>

Numerical results in Table (1) reveal that increase in confidence level

a) increases the sample size quite rapidly when the test time is short

b) has no significant effect on sample size when the test time is relatively longer and

c) introduces sharp growth in sample size when the shape parameter (\( \beta \)) increases with relatively short test time.

The effect of the change in the shape parameter \( \beta \) when \( \theta=2 \) and 3 with reference to the minimum sample size and the experiment time when the confidence level is 0.95, is given in fig.1.

![Fig.1. The sample size vs. Experiment time for SKSP-2 with \( \beta=2, c=1, f=0.333, P^*=0.95 \)](image-url)
This figure indicates that increase in the shape parameters increase the sample size rapidly for shorter experimenter time compared to the increase of the sample size for longer experimental time at the specified confidence level of the specified skip-lot acceptance sampling plans for life testing.

Operating Characteristics (Oc) Values
The performance of the sampling plan is evaluated with the operating characteristic values. The increase in probability of the acceptance signifies better is the quality of the product. Therefore, the operating characteristic values for the proposed plan with reference to the ratio \( m/m_0 \) of the true median life to the specified life are obtained. To compute the operating characteristic values the minimum sample size were taken from the Table (1). The OC values as a function of the ratio \( m/m_0 \) when \( P^*, \beta, \theta \) and \( a \) are presented in Table (2) corresponding to the selected Skip-lot sampling plan.

**TABLE (2). Oc Values At Specified Consumer’s Confidence Level For Sksp-2 With \( \theta=2 \), i=3.**

<table>
<thead>
<tr>
<th>( P^* )</th>
<th>( n )</th>
<th>( a )</th>
<th>( m/m_0 )</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>32</td>
<td>0.3</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.5</td>
<td>0.9994</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.7</td>
<td>0.9970</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>45</td>
<td>0.3</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.5</td>
<td>0.9989</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.7</td>
<td>0.9940</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is seen that for all the selected Skip-Lot sampling test plans the OC increases to one more rapidly to move from a lower value to a higher value of the median ratios. This happen due to the increase in the required sample size at higher shape parameters. Further, it is observed that

a) increase in confidence level decreases the operating characteristic values for a given \( m/m_0 \)
b) increase in confidence level decreases the operating characteristic values for a given multiplier of the specified time.

c) increase in the ratio of median values increases the operating characteristic values for any given \( a \) and any specified confidence level.

Minimum Median Ratios
The producer may be interested to know what will be the minimum product quality level to be given to the consumers in order to keep both the producer’s risk at the specified level and consumer’s confidence at the required level. At the producer’s risk of \( \alpha \), the minimum ratio \( m/m_0 \) can be obtained by solving

\[
P_a \geq 1 - \alpha
\]

where \( P_a \) is given by Equation (4). Table (3) are obtained to present the minimum median ratios to the specified life at the given consumer’s confidence level and test times corresponding to the specified producer’s risks.

**Table (3). Minimum Median Ratios To The Specified Life At \( \alpha=0.05 \), i=3.**
Minimum median ratios obtained for SKSP-2 when producer’s risk in less than or equal to 0.05 at the specified consumer’s confidence level reveal that increase in $\beta$ decrease and increase in $\theta$ the minimum median ratios at the given consumer’s confidence level and the specified time multiplier for any given $\beta$, $\theta$ and $a$ increase in consumer’s confidence level increases the minimum median ratio.

### Selection Of Life Test Plans

**Example.** Assume that the lifetime of the product under consideration follows generalized log-logistic distribution with shape parameters $\beta=1$, $\theta=2$ and acceptance number $c=1$. Suppose that the producer wants to establish that the true median life is greater than or equal to 2000 hours with the confidence level of 0.95. The experimenter wants to stop the experiment at 1200 hours. This leads to the experiment termination multiplier, $a=0.3$. For the problem under consideration Table (1) gives a skip-lot sampling plan with the minimum sample size $n=28$. This sampling plan is put into operation as follows:

Select a sample size $n=28$ items from the lot and put on test for 1200 hours and accept the lot if no failure occurs or otherwise, reject the lot.

The producer may be concerned with the acceptance probability as the quality improves because they want to minimize the producer’s risk. Suppose that the producer is interested in knowing what quality level will lead to the producer’s risk less than 0.05. This can be answered from Table (3). The minimum ratio for $\beta=1$, $\theta=2$, $P^*=0.90$ and $a=0.3$ with $c=1$ is 3.146. So, the true median required of the product should be at least 3146 hours.

### Conclusion

A Skip lot sampling plan for truncated life test is proposed in order to make a decision on the submitted lot under the assumption that the lifetime of the products follows a generalized log-logistic distribution, which is useful in system reliability analysis because its pattern of failure rate is quite versatile.

### References


