A Stochastic Model On The Time To Recruitment For A Single Grade Manpower System With Two Types Of Attrition Using Univariate Policy Of Recruitment

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Abstract
In this paper, the problem of time to recruitment is studied for a single grade manpower system with two types of attrition, one with higher attrition rate and the other with lower attrition rate, generated by an ordinary renewal process of inter-policy decision times, using univariate policy of recruitment based on shock model approach. A different probabilistic analysis is made to derive the analytical result for variance of the time to recruitment for different cases on the distribution of the threshold for the cumulative loss of manpower in the organization.

Keywords: Single grade manpower system, two types of attrition, inter decision times, ordinary renewal process, shock model approach, Univariate policy of recruitment and variance of time to recruitment.

Introduction
A marketing organization is not free from attrition due to exodus of personnel when the management takes policy decisions regarding pay, perquisites and work targets. This attrition which leads to loss of manpower will adversely affect the smooth functioning of the organization if it turns its blind eye towards compensating this loss by recruitment. Frequent recruitment is not advisable as it involves more cost. In view of this situation and from the fact that the depletion of manpower and the inter-decision times are probabilistic, the organization requires an appropriate recruitment policy to plan for recruitment. In [3],[4],[7],[8],[9],[11] and [12] the authors have studied the time to recruitment by considering different conditions on the loss of manpower, the inter-decision times and threshold for cumulative loss of manpower and using a univariate CUM policy of recruitment which states that recruitment is done whenever the cumulative loss of manpower crosses or exceeds a threshold. In all the above cited work, the analytical results for the moments of the time to recruitment are obtained through its distribution, using Laplace-Stieltjes transform. The method used in this paper is not only free from the conventional Laplace transforms method, but it directly estimates the moments of the time to recruitment. The objective of this paper is to obtain an analytical result for variance of the time to recruitment, an important performance measure in manpower planning, for different cases on the distribution of the threshold, using a different probabilistic analysis.

Model Description
Consider an organization taking policy decisions at random epochs in \((0,\infty)\). There are two types of policy decisions, one type with high attrition rate \(\lambda_1\) and the other with low attrition rate \(\lambda_2\). Let \(p\) and \((1-p)\) be the proposition of decisions with high and low attrition rate respectively. At every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For \(i=1,2,3,\ldots\), let \(X_i\) be the independent and identically distributed continuous random variables representing the amount of depletion of manpower(loss of man hours) due to the \(i^{th}\) policy decision with distribution \(G(.)\) and probability density functions \(g(.)\); \(S_i\) be the cumulative loss of manpower in the first \(i\) decisions; \(U_i\) the time between \((i-1)^{th}\) and \(i^{th}\) decisions, be independent and identically distributed hyper exponential random variables.

With mean \(E(U) = \frac{p}{\lambda_1} + \frac{(1-p)}{\lambda_2}\) and variance \(Var(U) = \frac{2p}{\lambda_1^2} + \frac{2(1-p)}{\lambda_2^2} - \left[\frac{p}{\lambda_1} + \frac{(1-p)}{\lambda_2}\right]^2\),

and \(R_i\), be the waiting time up to \((i+1)^{th}\) decisions. Let \(T\) be a continuous random variable representing the threshold for the cumulative loss of manpower with distribution \(H(.)\) and probability density functions \(h(.)\).Let \(\chi(A)\) be the indicator function of the event \(A\).Let \(W\) be the time to recruitment for the organization with mean \(E(W)\) and variance \(Var(W)\). The univariate CUM policy of recruitment employed in this paper is stated as follows: Recruitment is done whenever the cumulative loss of man hours in the organization exceeds \(T\).
Main Results
By the recruitment policy, recruitment is done whenever the cumulative loss of manpower exceeds the threshold T. When the first decision is taken, recruitment would not have been done for $U_1$ units of time. If the loss of manpower $X_t$ due to the first policy decision is greater than $T$, then recruitment is done and in this case $W = U_1 + R$. However, if $S_1 > T$, the non-recruitment period will continue till the next policy decision is taken. If the cumulative $S_2$ of the loss of manpower in the first two decisions exceeds $T$, then recruitment is done and $W = U_1 + U_2 + R$. If $S_2 > T$, then the non-recruitment period will continue till the next policy decision is taken and depending on $S_2 > T$ or $S_2 < T$, recruitment is done or the non-recruitment period continues and so on. Hence

$$W = \sum_{i=0}^{\infty} R_{i+1} \chi(S_i < T < S_{i+1})$$  \hfill (1)

From (1) and from the definition of $R_{i+1}$, we get

$$E(W) = E(U) \sum_{i=0}^{\infty} (i+1) \chi(S_i < T < S_{i+1})$$ \hfill (2)

and

$$E(W^2) = \sum_{i=0}^{\infty} \left[ E(U) + (i+1)(E(U))^2 \right] \chi(S_i < T < S_{i+1})$$ \hfill (3)

By the law of total probability, we get

$$P(S_i < T < S_{i+1}) = \int_{0}^{T} G(\tau) \left( e_{S_2}(x, \tau) \right) dx dt$$ \hfill (4)

Using (4) in (2) and (3) on simplification we get

$$E(W) = E(U) \int_{0}^{T} \int_{0}^{\infty} \left( e_{S_2}(x, \tau) \right) dx dt$$ \hfill (5)

and

$$E(W^2) = \text{Var}(U) \sum_{i=0}^{\infty} (i+1) \int_{0}^{T} \int_{0}^{\infty} \left( e_{S_2}(x, \tau) \right) dx dt$$ \hfill (6)

Therefore,

$$\text{Var}(W) = \text{Var}(U) \sum_{i=0}^{\infty} (i+1) \int_{0}^{T} \int_{0}^{\infty} \left( e_{S_2}(x, \tau) \right) dx dt$$

Special Case
We now obtain explicit analytical expressions for $E(W)$ and $\text{Var}(W)$ by assuming $G(x) = 1 - e^{-ax}$ and considering several cases on different distributions for the threshold.

Suppose the distribution of the threshold is exponential with parameter ‘$\theta$’. In this case, from (5) and (7), we get

$$E(W) = \left( \frac{\alpha + \theta}{\theta^2} \right) E(U)$$ \hfill (9)

and

$$\text{Var}(W) = \left( \frac{\alpha + \theta}{\theta^2} \right) \left[ \text{Var}(U) + \left( \frac{\alpha}{\theta^2} \right) (E(U))^2 \right]$$ \hfill (10)

(9) and (10) give the mean and variance of the time to recruitment for case(i) where $E(U)$ and $\text{Var}(U)$ are given by (8).

Case(ii)
Suppose the distribution of the threshold is an extended exponential with scale parameter ‘$\theta$’ and shape parameter 2. In this case, from [5],

$$h(t) = \left( 2\theta e^{-\theta t} \left( 1 - e^{-\theta t} \right) \right)$$

Therefore from (5) and (7) and on simplification, we get

$$E(W) = \left[ 2 \left( \frac{\alpha + \theta}{\theta^2} \right) - \left( \frac{\alpha + 2\theta}{2\theta^2} \right) \right] E(U)$$ \hfill (11)
and
\[
\text{Var}(W) = \left( \frac{e^{2\mu}}{\theta} - \frac{e^{2\mu}}{2\theta} \right)\text{Var}(U) + \left( \frac{e^{2\mu}}{\theta} - \frac{e^{2\mu}}{2\theta} \right)^2 \langle E(U) \rangle^2
\]  
(12)

(11) and (12) give the mean and variance of the time to recruitment for case (ii) where \( E(U) \) and \( \text{Var}(U) \) are given by (8).

**Case (iii)**

Suppose the distribution of the threshold has SCBZ property with parameter \( \theta_1 \) and \( \theta_2 \). In this case, from [9],
\[
h(t) = p_1(\theta_1 + \mu)e^{-(\theta_1 + \mu)t} + q_1(\theta_2 - \mu)e^{-\theta_2 t}
\]
Therefore from (5) and (7) and on simplification, we get
\[
E(W) = \left[ p_1 \left( \frac{e^{2\mu} - e^{\theta_1 + \mu}}{\theta_1 + \mu} \right) + q_1 \left( \frac{e^{2\mu} - e^{\theta_2}}{\theta_2} \right) \right] E(U)
\]  
(13)
and
\[
\text{Var}(W) = \left[ p_1 \left( \frac{e^{2\mu} - e^{\theta_1 + \mu}}{\theta_1 + \mu} \right) + q_1 \left( \frac{e^{2\mu} - e^{\theta_2}}{\theta_2} \right) \right] \text{Var}(U)
\]  
\[+ \left( \frac{e^{2\mu} - e^{\theta_1 + \mu}}{\theta_1 + \mu} - \frac{e^{2\mu} - e^{\theta_2}}{\theta_2} \right) \langle E(U) \rangle^2
\]  
(14)

(13) and (14) give the mean and variance of the time to recruitment for case (iii) where \( E(U) \) and \( \text{Var}(U) \) are given by (8).

**Conclusion**

From the results for the performance measures, we note that the mean and variance of the time to recruitment for all the three cases decrease or increase according as the parameter \( p \) or \( \alpha \) increases, keeping other parameters fixed and these conclusions agree with reality. In the presence of attrition, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. Further, the observations on the performance measures given in this paper will be useful to enhance the facilitation of the assessment of the manpower profile in future manpower development prediction, not only on industry but also in a broader domain.

**References**